PHYSICS NYB-10/11 Winter 2007

Lecture 9: Electrical current and resistance

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Review

- Matter is made up of positively charged, negatively charged and neutral particles
- Opposite charges attract and like charges repel according to Coulomb's law $|\vec{F}| = k_e \frac{|q_1||q_2|}{r^2}$
- A charge q sets up an electric field $\vec{E} = k_e \frac{q}{r^2}\hat{r}$ and an electric potential $V = k_e \frac{q}{r}$.
- The electric field is the slope of the electric potential $E_r = -\frac{\partial V}{\partial r}$

Review

- A charge q_0 placed in an electric field $\vec{E}$ feels a force $\vec{F} = q_0\vec{E}$
- The potential energy from placing a charge q_0 in an electric potential V is $U = q_0V$
- It is now time to put all of these concepts to good use; where have you seen potential V before?
- You learned in high-school that in electrical circuits,

$$\boxed{V = RI}$$

- We now have all the tools required to *understand* this law

Recall: conductors in electric fields

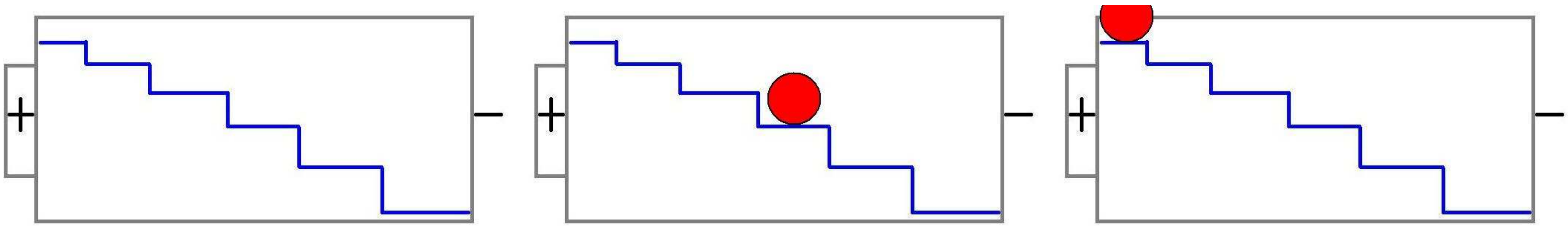
- Remember, in a conductor charges are free to move, so if there's an electric field inside a conductor, charges are accelerated
- What we saw is that the charges creat their own electric field which cancels the original one
- The electric field inside a conductor at electro-static equilibrium is zero.

Recall: conductors in electric fields

- But what if we had a device that could prevent equilibrium from being reached?
- We need something to *maintain the electric field* in the conductor.
- In other words, we want to maintain a *potential difference* inside the conductor.
- Does such a device exist???

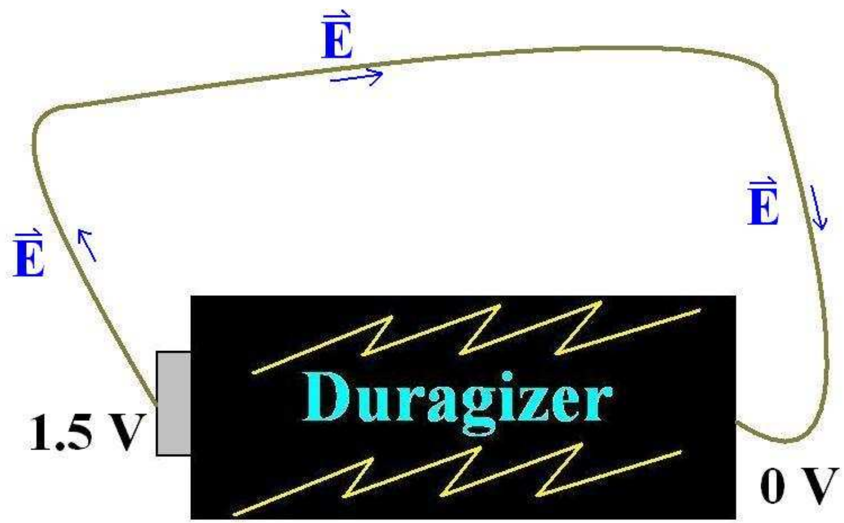
Batteries

- A battery is a source of constant potential difference
- It takes charges and does work on them, giving them potential energy
- This energy comes from chemical energy in the battery, which is limited; batteries eventually run out of *energy*
- Batteries don't run out of current!!! They are not a source of electrons!!! They give potential energy to electrons *already in the wires*.



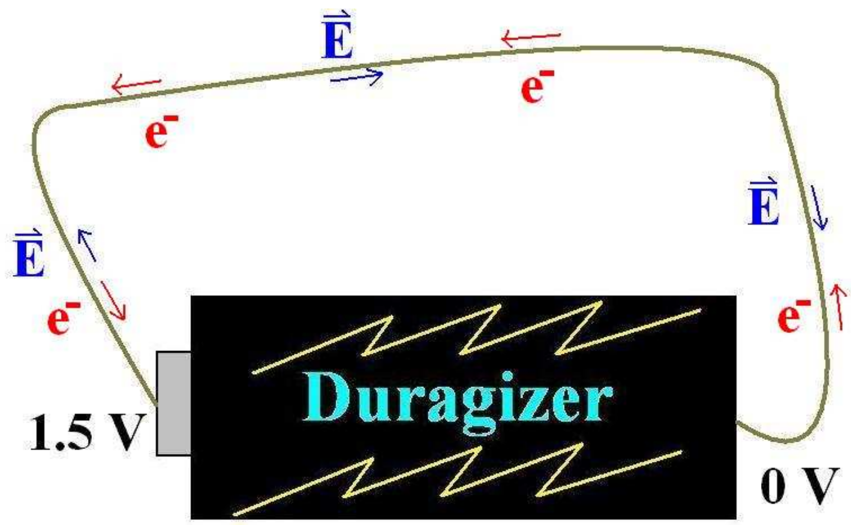
Electrical current

- We connect a conducting wire to the two ends of a battery.
- There is now a potential difference between the ends of the wire.
- There is therefore an electric field throughout the wire.



Electrical current

- Electrons move in the wire from low potential to high potential, and the battery then brings them back to their starting point, so the process doesn't stop
- Equilibrium is *not reached*. There is a constant *current* in the wire.



Electrical current

Sort of like this...

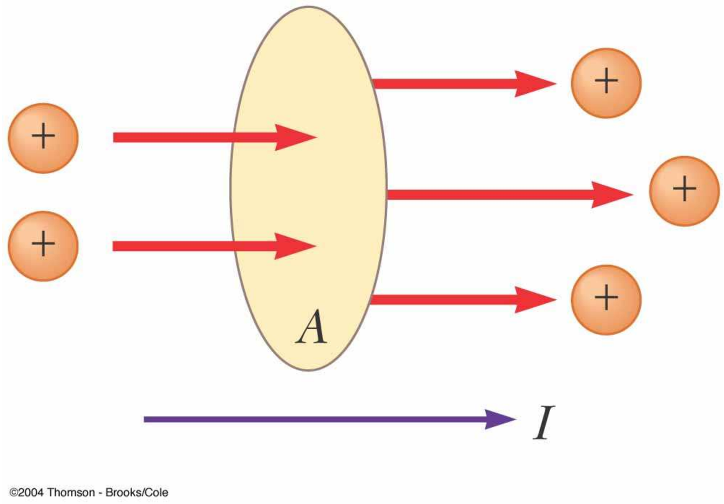
Electrical current

Or like this...

- Just like in a battery, the chairlift is not a source of skiers
- The chairlift brings the skiers from one height to another
- The skiers go down the hill, but are constantly brought back up by the lift
- There is a constant flow, a current, of skiers down the hill.

Electrical current

More rigourously, current is defined as *the rate at which charges flow through a given surface*.



When an amount of charge ΔQ flows through a surface during a time Δt, the current through the surface is

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

Electrical current

If we take the limit where $\Delta t \rightarrow 0$, we get instantaneous current

$$I = \frac{dQ}{dt}$$

What are the units of current?

Amperes (A) which are Coulombs per second (C/s).

Electrical current

Things that are good to know

- In a metal, negatively charged electrons move and create a current
- In general, current can come from the movement of both positive and/or negative charges
- The direction of current is defined as the direction in which *positive* charge flows
- Therefore the direction of current in a metal wire is actually opposite the physical flow of charge
- *Current is conserved*. What goes in at a given point must also come out at the same point! (Or else charge would accumulate there).

Example

Evil Dr.Cos has put the finishing touches on his greatest creation: Frankeinstein! (He plans to use this creature to achieve his fiendish plan of forcing health science students everywhere to switch to pure and applied...) In order to bring Frankeistein to life, he passes a current of 1000 A going from the red wire through Frankeinstein and then out the blue wire for a time of 1 minute. What direction is the electric field pointing in? In what wire is the potential greatest? How many electrons will pass through Frankeinstein? What direction are they heading in?

Example

The direction of current is defined as the direction of motion of positive charges. To get positive charges to move from the red to the blue wire, the electric field has to point from the red to the blue wire.

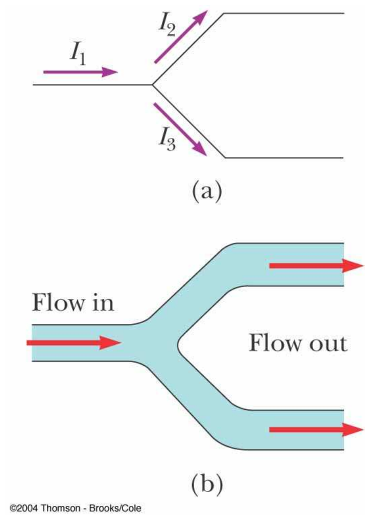
The electric field always points in the direction of decreasing potential, therefore the red wire must be at higher potential than the blue wire.

The current is 1000 A, meaning 1000 C/s during 60 seconds. So overall, $60\ 000$ C pass through Frankeistein. We know that one electron has a charge of magnitude 1.6×10^{-19} C, so the number of electrons is

$N_e = 60\ 000/(1.6 \times 10^{-19}) = 3.75 \times 10^{23}$ electrons.

The current is heading from the red to the blue wire, which means that the electrons are heading from the blue to the red, since they are negative.

Conservation of current

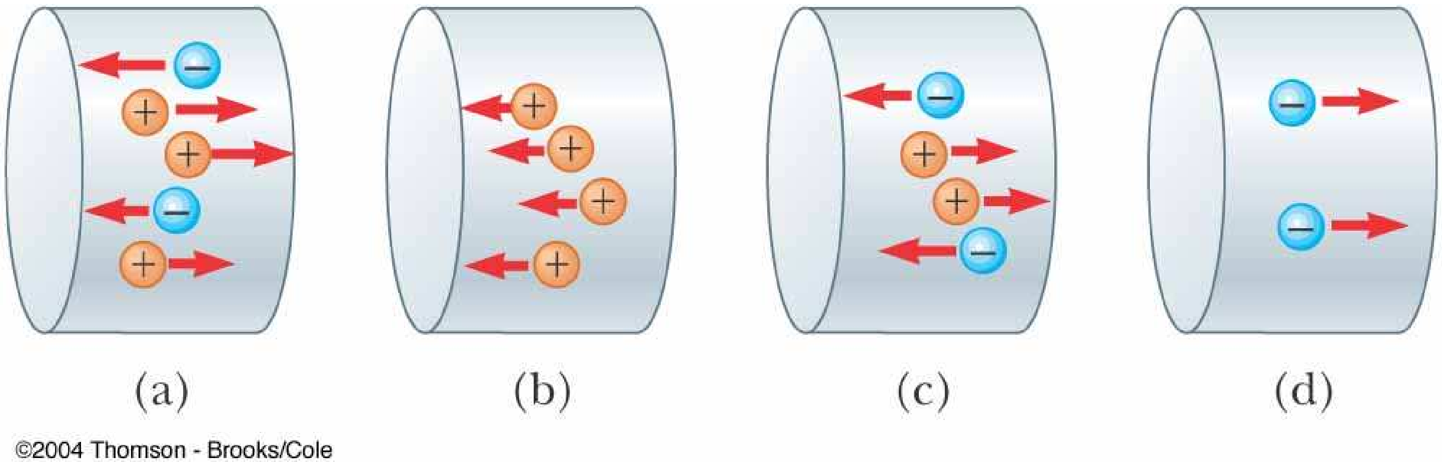


If $I_1 = 10$ A, and $I_2 = 7$ A, what's I_3

You have 10 C coming into the junction point every second. There must be 10 C coming out of the junction every second, or else charges would be accumulating there. If 7 C/s are coming out through the top wire, then there must be 3 C/s coming out of the bottom one, so $I_3 = 3$ A.

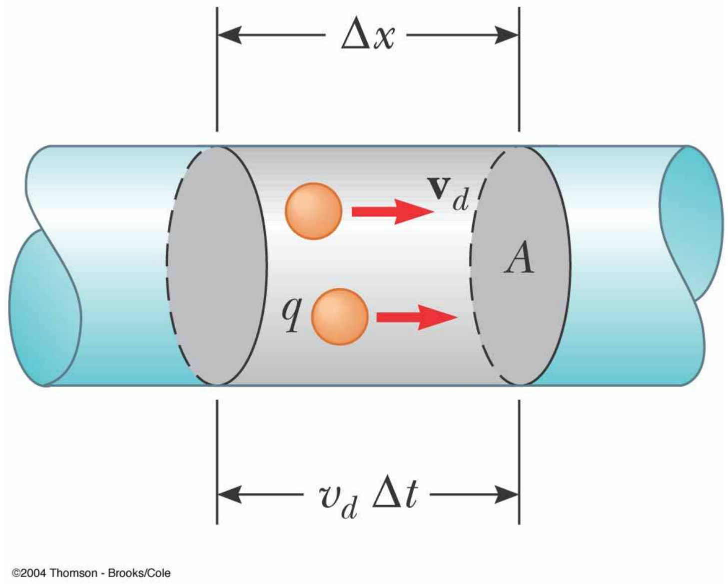
Electrical current

The pictures below show electric charges (each $+q$ or $-q$) crossing a surface in a fixed amount of time Δt. What is the current and in what direction is it flowing in each case?



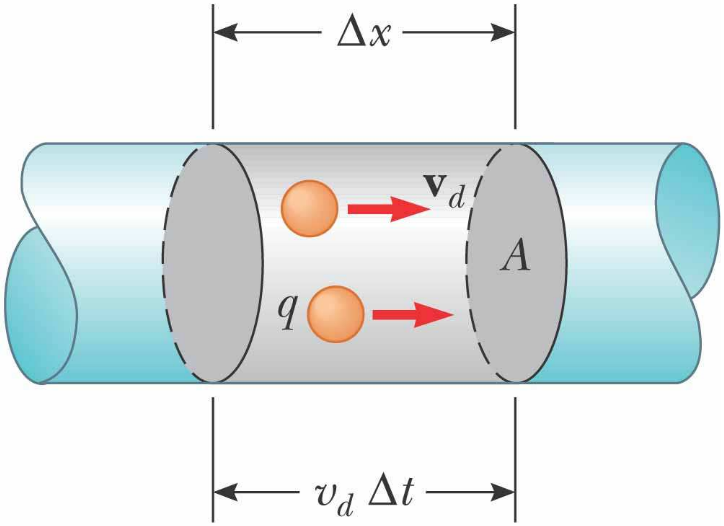
a) $I = 5q/\Delta t$ to the right
b) $I = 4q/\Delta t$ to the left
c) $I = 4q/\Delta t$ to the right
d) $I = 2q/\Delta t$ to the left

Electrical current



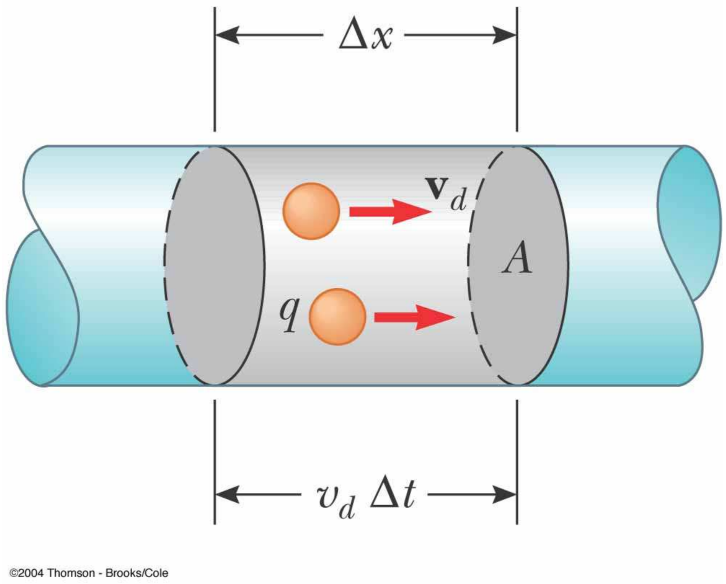
Imagine we have a surface A inside a certain material, and we want to know how many charges are going to cross it in a certain time Δt. How do we figure this out?

Electrical current



If the charges have a speed v_d towards the surface, then in a time Δt any charge a distance $\Delta x \leq v_d \Delta t$ away will cross the surface. How many charges does this make?

Electrical current



The volume from which the charges can come is $A\Delta x$. If there are n charge carriers per unit volume which each carry a charge q, the total amount of charge is $\Delta Q = nqA\Delta x = nqAv_d\Delta t$. So

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nqAv_d$$

Drift speed

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nqAv_d$$

In this expression, v_d is called the *drift speed* of the charge carriers. It is the average speed at which charges actually travel inside a medium. Let's estimate a typical drift speed.

Take a typical copper wire with a cross section of 3.31×10^{-6} m^2. What is the drift speed of electrons when the wire carries a typical current of 10 A, knowing the density of copper is 8.95 g/cm^3? How long does it take for an electron to move from a light switch to a light bulb through 3 meters of wiring?

Drift speed

The molar mass of copper is 63.5 g/mol. So one mol of copper occupies a volume $V = \frac{m}{\rho} = 7.09$ cm^3. One mol is 6.02×10^{23} atoms, each of which has one free electron that can move around in the wire. So the charge density $n = 8.49 \times 10^{28}$ electrons/m^3. Now we know that $I = nqAv_d$ so $v_d = \frac{I}{nqA} = 2.22 \times 10^{-4}$ m/s.

It will therefore take $t = \Delta x / v_d =$3 hours and 45 minutes.

Drift speed

Wait a minute...

Does it take over 3 hours for the lights to turn on when you turn a switch?

No...

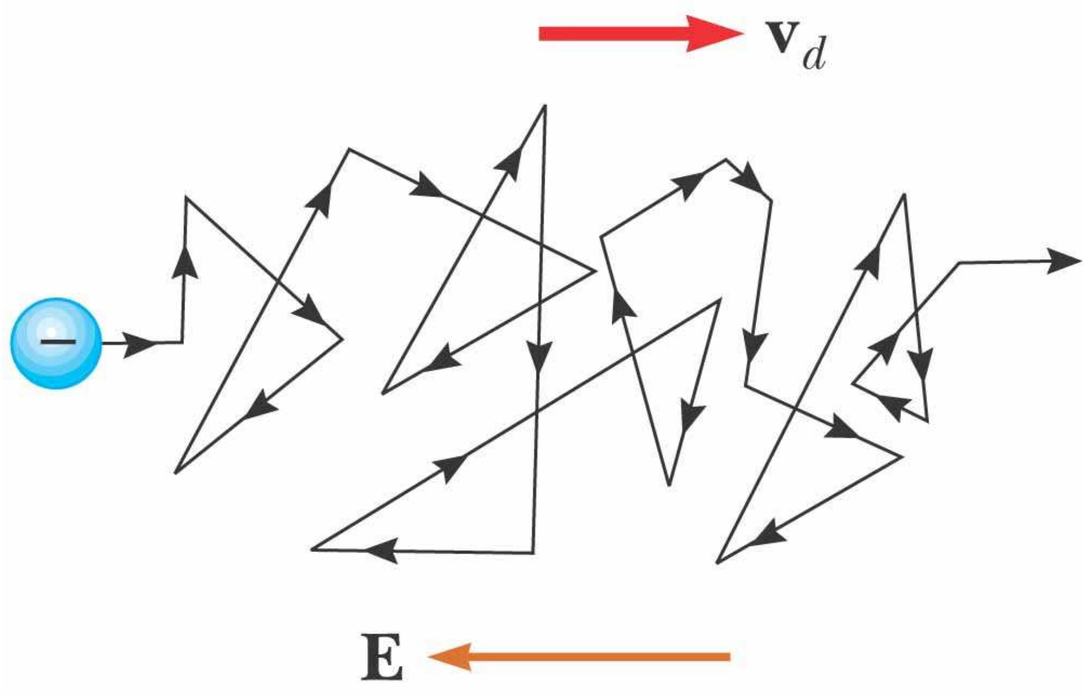
The resolution to this apparent paradox is that when you turn the switch, the *electric field* travels at almost the speed of light, so that electrons **everywhere in the wire** almost immediately begin to move (very slowly), so there is current everywhere right away!

Drift speed

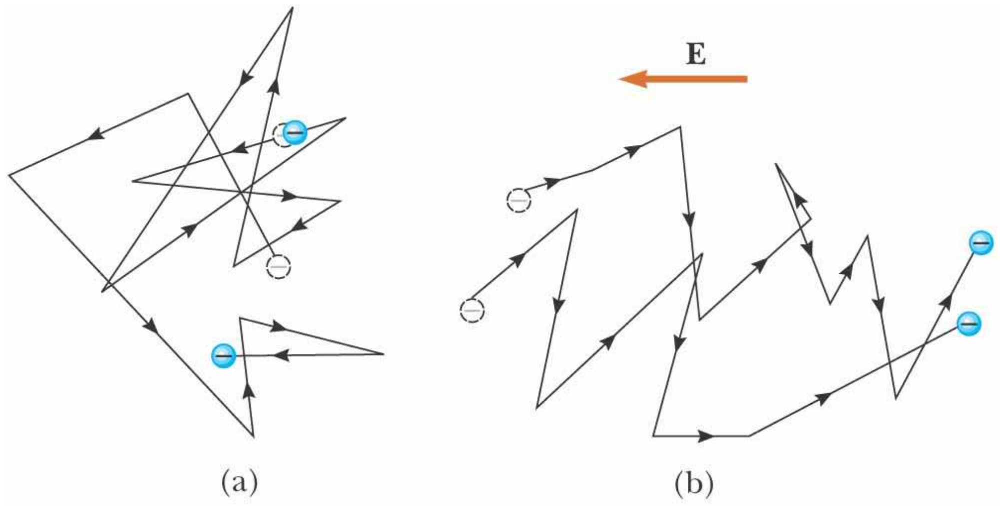
Why is drift speed so low? Naively, a 120 V potential difference over a 3 m wire leads to an electric field of magnitude 40 V/m, or 40 N/C. The charge of an electron is -1.6×10^{-19} C, so the magnitude of the force on the electron is 6.4×10^{-18} N, which for a mass of 9.1×10^{-31} kg should give an acceleration of 7×10^{12} m/s^2! It should take $t = \sqrt{2\Delta x/a} = 9 \times 10^{-7}$ seconds for the electron to travel that distance... What's going on?

Drift speed

First off, there isn't such a potential difference in the wire, since most of the potential drop occurs in the lightbulb where the resistance is large. Even so, the electron should not take so long to travel this distance. What happens is that the metal is full of atoms. The electron repeatedly collides with other particles on its way toward the lightbulb.



Drift speed



$$\vec{v}_f = \vec{v}_i + \vec{a}t; \quad \vec{v}_f = \vec{v}_i - \frac{e\vec{E}}{m_e}t$$

After a collision, $\vec{v}_i$ is essentially random. The electron then accelerates for a short while, with acceleration $\vec{a} = -\dfrac{e\vec{E}}{m_e}$ so that overall, its average velocity is $\vec{v}_{avg} = \vec{v}_d = -\frac{e\vec{E}}{m_e}\tau$, where τ is the average time it gets to accelerate before undergoing another collision.

Resistance: Ohm's law

Current density: We have just seen the current is given by the expression $I = nqAv_d$. We will now define current *density* as the current per unit area

$$J = \frac{I}{A} = nqv_d$$

More generally, we can write the current density as a vector

$$\vec{J} = nq\vec{v}_d = nq\left(\frac{q}{m}\vec{E}\tau\right) = \left(\frac{nq^2\tau}{m}\right)\vec{E}$$

We are now ready to state Ohm's law

$$\vec{J} = \frac{1}{\rho}\vec{E} = \sigma\vec{E}$$

Resistance: Ohm's law

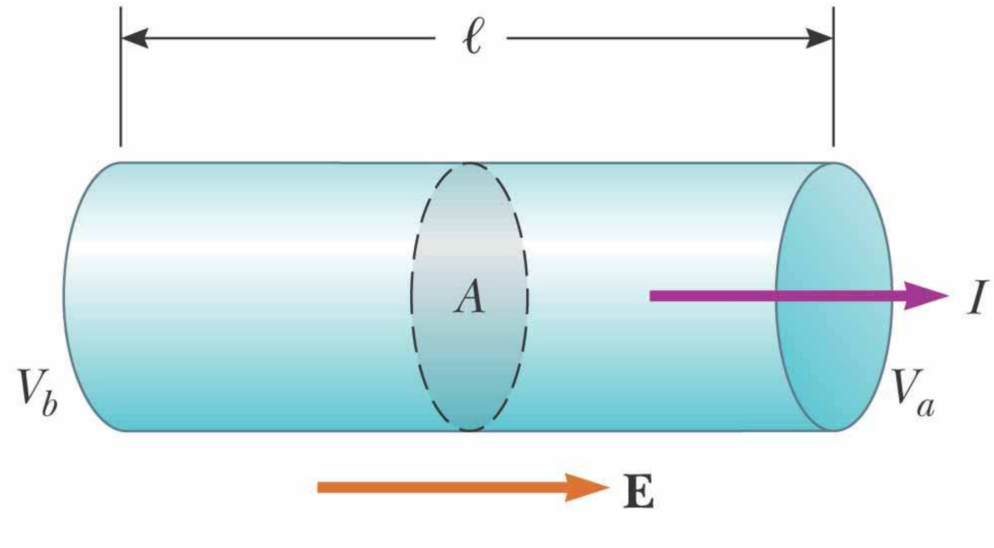
$$\boxed{\vec{J} = \frac{1}{\rho}\vec{E} = \sigma\vec{E}}$$

Here, σ and ρ are called, respectively, conductivity and resistivity. They are properties of a given material.

You should now be thinking "Isn't Ohm's law $\Delta V = RI$???"

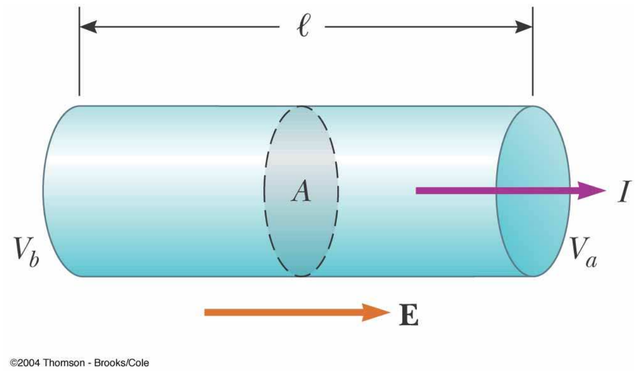
Ohm's law???

Actually, the form you probably all know and love is a *consequence* of the *real* Ohm's law. Let's see how this works.



Take a wire of length l and area A. Apply a potential difference ΔV between its ends. We assume the electric field is constant, so $E = \dfrac{\Delta V}{l}$ V/m. So $J = \sigma E = \sigma \dfrac{\Delta V}{l}$.

Resistance



$$
\begin{aligned}
J &= \sigma \frac{\Delta V}{l} \\
\Delta V &= \frac{Jl}{\sigma} = \frac{Il}{A\sigma} = \left(\frac{l}{A\sigma} \right) I \\
\Rightarrow \Delta V &\equiv RI
\end{aligned}
$$

where $R = \frac{l}{A\sigma} = \rho \frac{l}{A}$ is the *resistance* of a wire of length l, area A and resistivity ρ. Resistance is measured in *ohms*, Ω.

Resistance

$$\boxed{R = \frac{l}{A\sigma} = \rho\frac{l}{A}}$$

- $\sigma = \dfrac{1}{\rho}$ depends on the material the wire is made of
- A longer wire has more resistance, since the charges must travel through more material to get through it
- A fatter wire has less resistance, since there is “more room” for the charges to move
- The resistance does *not* depend on the applied potential difference (the magnitude of the electric field)
- This last statement is true for *ohmic* materials, where I depends linearly on ΔV

Resistance and temperature

$$I = \frac{\Delta V}{R}$$

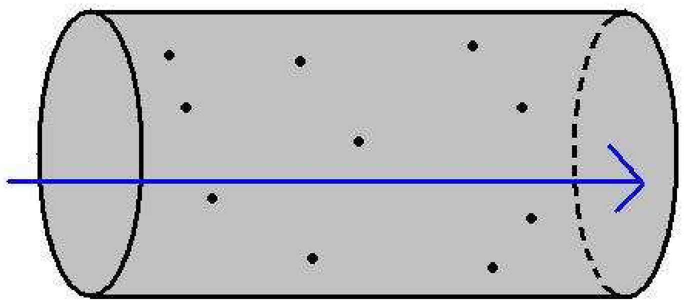
In an ohmic material, this law holds. R depends on the properties of the resistor only; it is not a dependent variable!!!

Resistance does vary with temperature, according to

$$R = R_0 \left[1 + \alpha(T - T_0)\right]$$

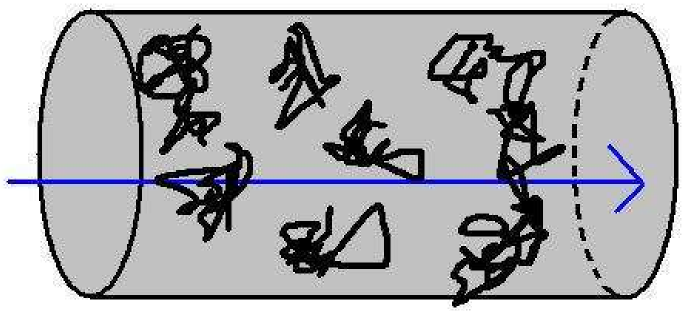
How can we understand this behaviour?

Resistance and temperature



Resistance depends on the number of collisions a charge will undergo when crossing the length of the material. At low temperatures, the atoms in the material don't move much.

Resistance and temperature



At higher temperatures, the atoms jiggle around randomly, and effectively take up more space than they did at low temperature. This increases the probability that a charge will collide with atoms, and therefore decreases the charges' drift speed, thus increasing resistance.

What to read for next lecture

- 27.6, 28.1